

Lattice QCD study of Nucleon structure

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Nucleon Structure on a Lattice

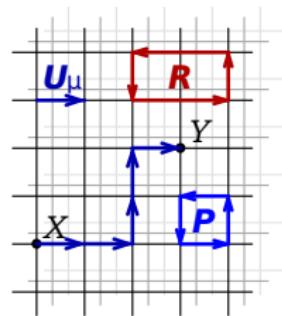
Numerical Feynman integration

$$\langle \mathcal{O} \rangle = \int DUD\psi D\bar{\psi} \mathcal{O}[U, \psi, \bar{\psi}] e^{-S[U, \psi, \bar{\psi}]} \rightarrow \frac{1}{N} \sum^N \tilde{\mathcal{O}}[U]$$

- Euclidean QFT: $\begin{cases} x^0 \equiv t & \rightarrow -ix_4 \equiv -i\tau \\ p^0 \equiv E & \rightarrow ip_4 \\ \langle N(t)\bar{N}(0) \rangle & \rightarrow e^{-E\tau} \end{cases}$
- Fields on a discrete space-time grid:

$$A_\mu^a(x) \rightarrow U_{x,\mu} = \mathcal{P}e^{-i \int_x^{x+\hat{\mu}} dx \cdot (A^a \frac{\lambda^a}{2})}$$

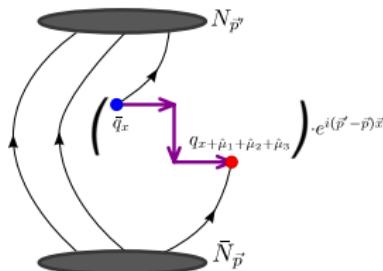
$$S_g[A_\mu] \sim (F_{\mu\nu}^a)^2 \rightarrow A \text{Tr}(\textcolor{blue}{P}) + B \text{Tr}(\textcolor{red}{R})$$



Lightest pion masses $m_\pi \approx 150, 200, 250$ MeV in this work:
(generated by the BMW collaboration)

- Symanzik $\mathcal{O}(a^2)$ -improved gauge action
- Clover-improved Wilson action (HEX² gauge links)
- Dynamical 2+1 quarks, $m_u = m_d$; m_s tuned close to physical
- Lattice spacing $a = 0.116$ fm (+ one ens. with $a = 0.091$ fm)

Quark Bilinear Operators (Twist-2)



Mellin moments of GPDs \iff symmetric, trace = 0 quark operators:

- In continuum: Lorentz symmetry preserves operators from mixing
- On a lattice: Hypercubic group has 20 irreducible representations

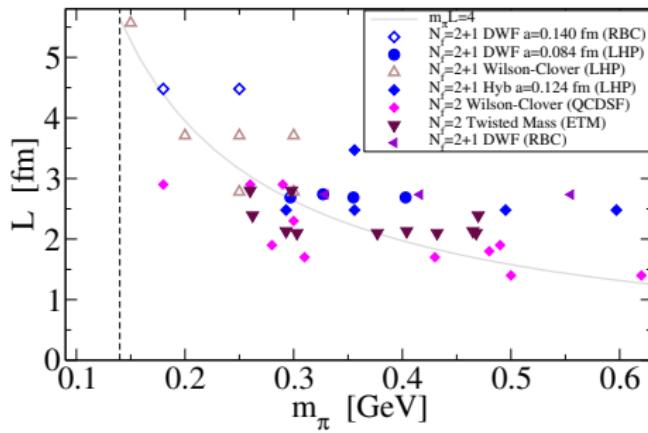
$n = 1$	$\bar{q}\gamma_\mu q$	$\rightarrow \mathbf{4}_1^-$
$n = 2$	$\bar{q}[\gamma_{\{\mu} i\overset{\leftrightarrow}{D}_{\nu\}} - \langle \text{Tr} \rangle]q$	$\rightarrow \mathbf{3}_1^+ \oplus \mathbf{6}_3^+$
$n = 3$	$\bar{q}[\gamma_{\{\mu} i\overset{\leftrightarrow}{D}_{\nu} i\overset{\leftrightarrow}{D}_{\rho\}} - \langle \text{Tr} \rangle]q$	$\rightarrow \mathbf{8}_1^- \oplus \mathbf{4}_1^- \oplus \mathbf{4}_2^-$
$n = 4$	$\bar{q}[\gamma_{\{\mu} i\overset{\leftrightarrow}{D}_{\nu} i\overset{\leftrightarrow}{D}_{\rho} i\overset{\leftrightarrow}{D}_{\sigma\}} - \langle \text{Tr} \rangle]q$	$\rightarrow \mathbf{1}_1^+ \oplus \mathbf{3}_1^+ \oplus \mathbf{6}_3^+ \oplus \mathbf{2}_1^+ \oplus \mathbf{1}_2^+ \oplus \mathbf{6}_1^+ \oplus \mathbf{6}_2^+$ [Göckeler et al, Phys.Rev.D54,5705(1996)]
...		

Mixing coefficients $\sim \Lambda_{\text{UV}}^{d_1-d_2} = \left(\frac{1}{a}\right)^{d_1-d_2}$ limit $n \leq 4$ in practice

Lattice QCD

Solving QCD numerically is hard because

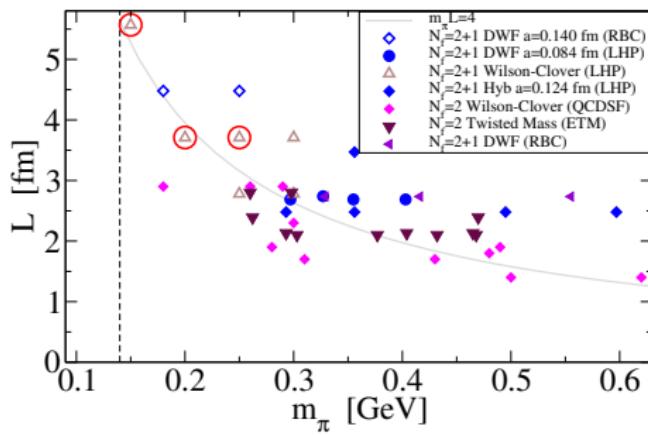
- light quarks are expensive: cost $\sim \frac{1}{m_\pi}$
- need large physical size of the box $L \gtrsim \frac{4}{m_\pi}$
- have to take continuum limit $a \rightarrow 0$, $L_{\text{lat}} = \frac{L}{a} \rightarrow \infty$
- chiral symmetry is expensive to preserve in lattice regularization



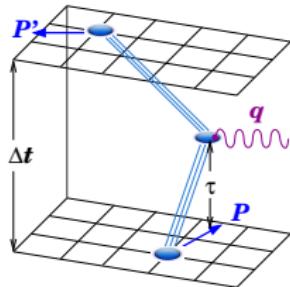
Lattice QCD

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Contribution from Excited States



$$\langle N(\Delta t, \vec{x}) \mathcal{O}(\tau, \vec{y}) \bar{N}(0) \rangle \longrightarrow \langle P' | \mathcal{O} | P \rangle \\ (\Delta t, \tau, (\Delta t - \tau) \rightarrow \infty)$$

Bias from excited states $\bar{N}_{\text{lat}} |\Omega\rangle = |N\rangle + C|X\rangle$

$$\langle N | \mathcal{O} | N \rangle_{\text{lat}} \cong \langle N | \mathcal{O} | N \rangle$$

$$+ |C|^2 \langle X | \mathcal{O} | X \rangle e^{-\Delta E_X \cdot \Delta t}$$

$$+ C \langle N | \mathcal{O} | X \rangle e^{-\Delta E_X \cdot \tau}$$

$$+ C \langle X | \mathcal{O} | N \rangle e^{-\Delta E_X (\Delta t - \tau)}$$

Methods to isolate ground state m.e.:

- Multi-exponential fits
- *Generalized pencil of functions (GPoF)* [C. Aubin, K. Orginos, 1010.0202]
- “Summation method”,

$$\sum_{\tau} \langle N | \mathcal{O}(\tau) | N \rangle_{\text{lat}} \longrightarrow \langle P' | \mathcal{O} | P \rangle \cdot \Delta t + O(e^{-\Delta E_X \cdot \Delta t}) \cdot \Delta t + \text{const}, \\ \text{suppress } O(e^{-\Delta E_X \cdot \Delta t/2}) \rightarrow O(e^{-\Delta E_X \cdot \Delta t})$$

With noisy data, summation is the most stable

Electromagnetic Form Factors

Electromagnetic current in the nucleon:

$$\langle P' | \bar{q} \gamma^\mu q | P \rangle = \bar{U}(P') \left[F_1^q(Q^2) \gamma^\mu + F_2^q(Q^2) \frac{i \sigma^{\mu\nu} q_\nu}{2M_N} \right] U(P),$$

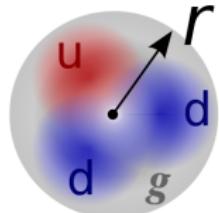
where $q = P' - P$ and $Q^2 = -q^2$

"Radii"

$$F(Q^2) \approx F(0) \left(1 - \frac{1}{6} \langle \mathbf{r}^2 \rangle \cdot Q^2 + O(Q^4) \right)$$

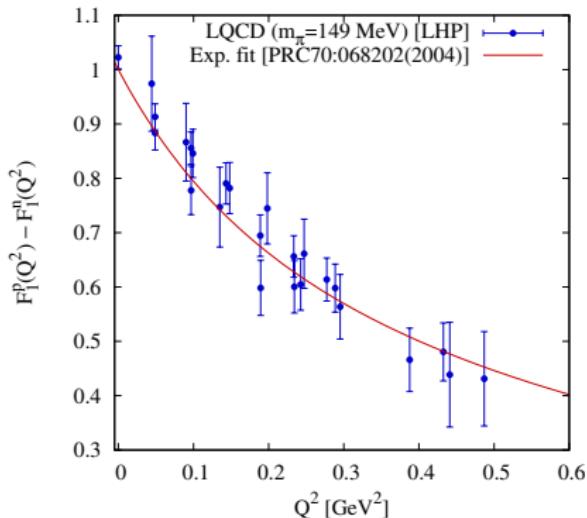
Revived experimental interest (JLab, Mainz, ...)

- proton radius controversy
- scaling at large Q^2 ($\gtrsim 1$ GeV 2)
- individual quarks (u, d) contributions to the structure

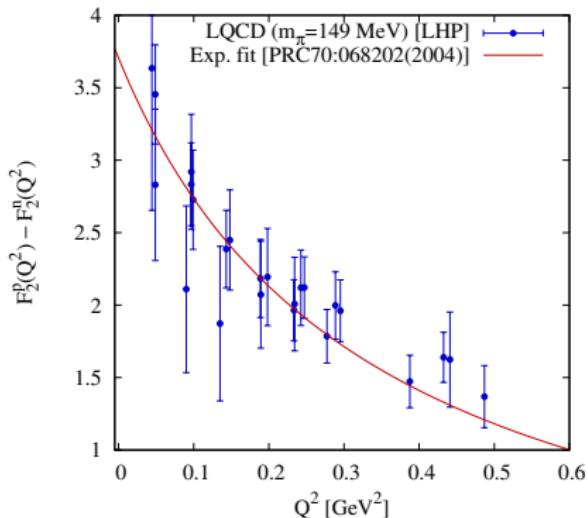


Nucleon (Isoscalar) Form Factors

Dirac form factor

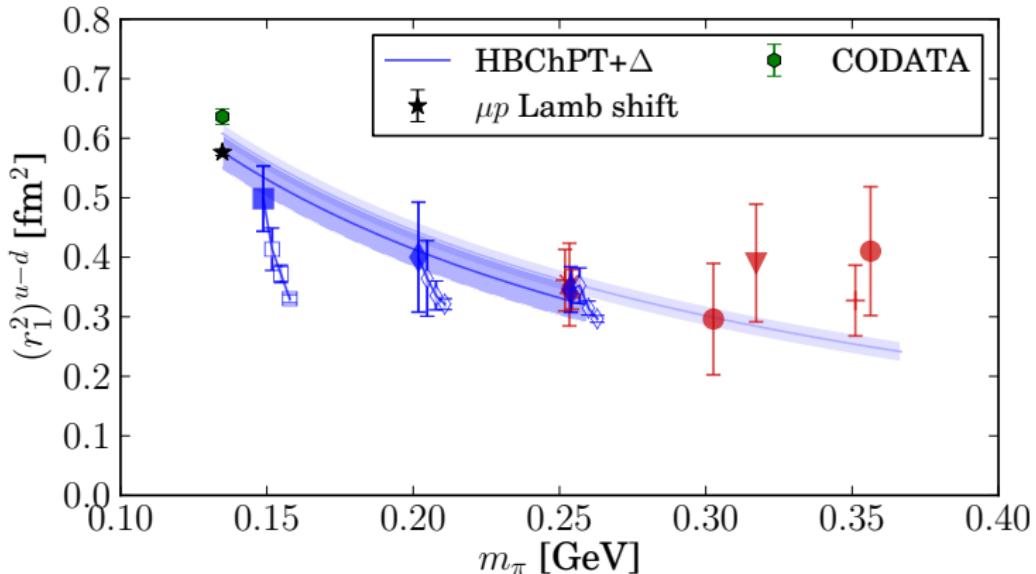


Pauli form factor



$$\langle P' | \bar{q} \gamma^\mu q | P \rangle = \bar{U}(P') \left[F_1^q(Q^2) \gamma^\mu + F_2^q(Q^2) \frac{i \sigma^{\mu\nu} q_\nu}{2M_N} \right] U(P),$$

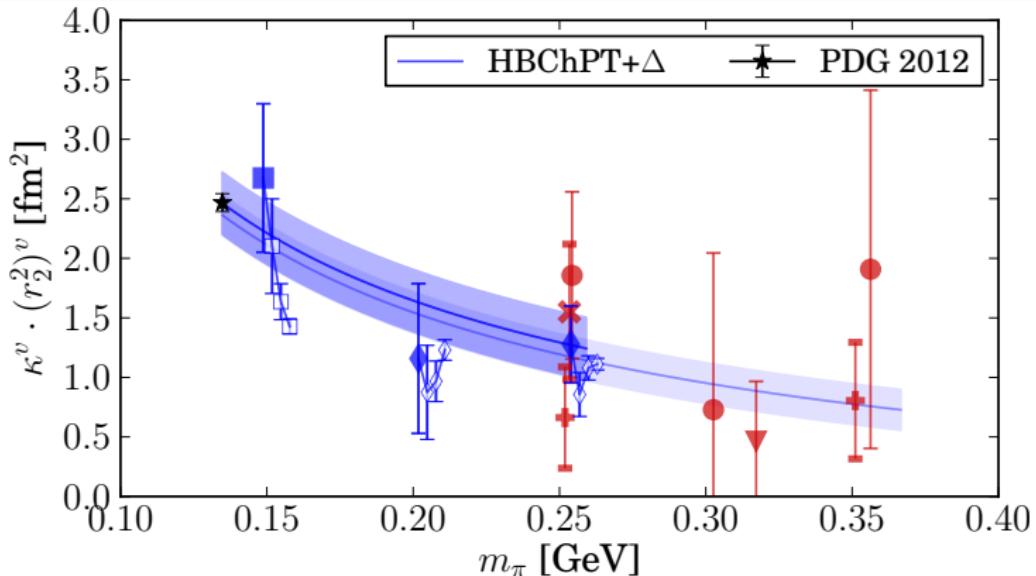
Nucleon (Isoscalar) Dirac Radius



$$F_1(Q^2) = 1 - \frac{1}{6} \langle r_1^2 \rangle \cdot Q^2 + O(Q^4)$$

- uncertainty comparable to the experimental discrepancy
- high sensitivity to pion cloud: corrections $\sim \log m_\pi$

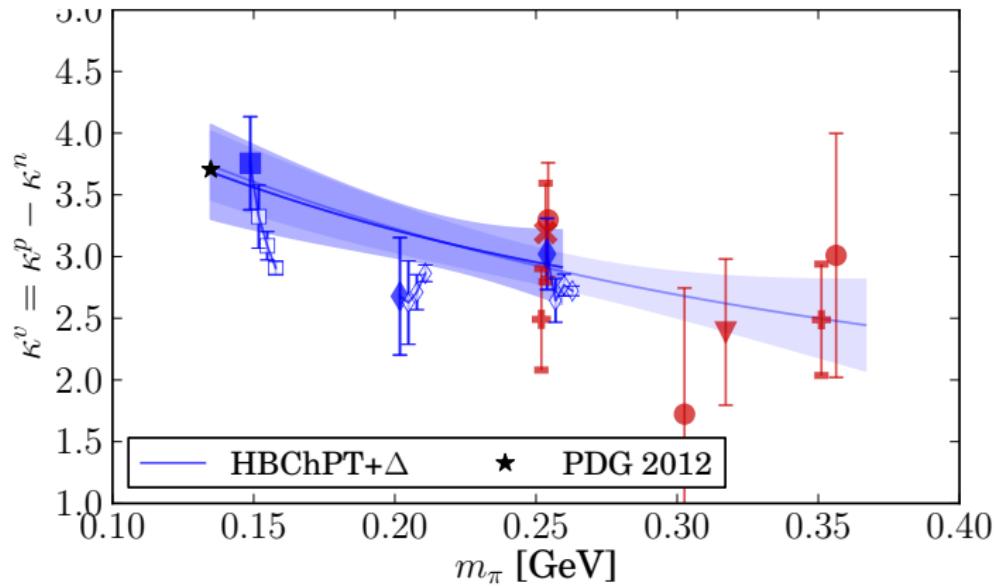
Nucleon (Isoscalar) Pauli Radius



$$F_2(Q^2) = \kappa_v \left(1 - \frac{1}{6} \langle r_2^2 \rangle \cdot Q^2 + O(Q^4) \right)$$

- precision limited by extrapolation from $Q_{\min}^2 \approx \left(\frac{2\pi}{L_s}\right)^2$,
- high sensitivity to pion cloud: corrections $\sim \frac{1}{m_\pi}$

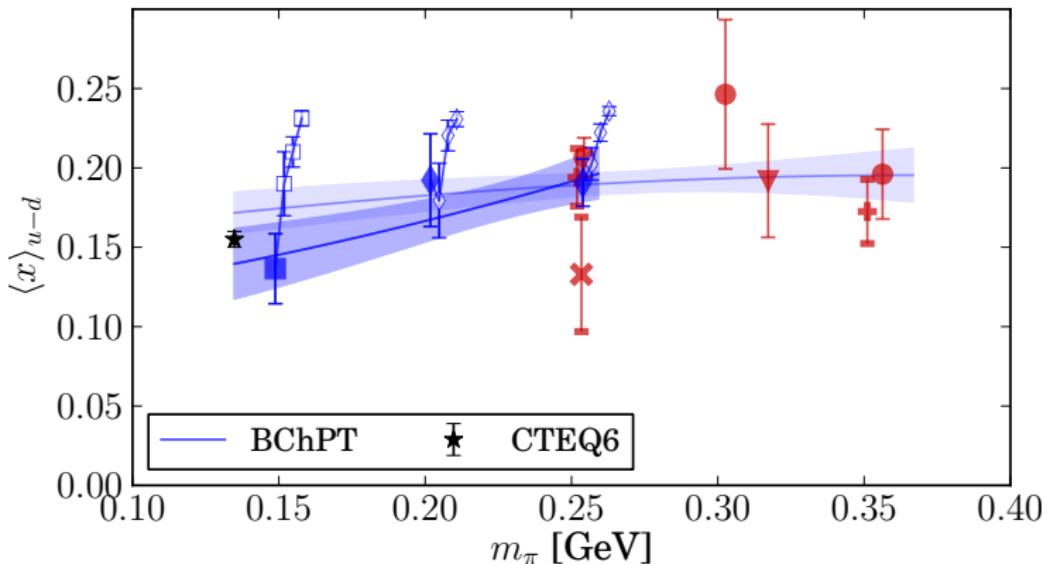
Nucleon (Isoscalar) Anom. Magnetic Moment



$$\kappa_v = F_2(0), \quad \mu = Q + \kappa$$

- precision limited by extrapolation from $Q_{\min}^2 \approx \left(\frac{2\pi}{L_s}\right)^2$,

Momentum Fraction Carried by Quarks ($\langle x \rangle_{u-d}$)

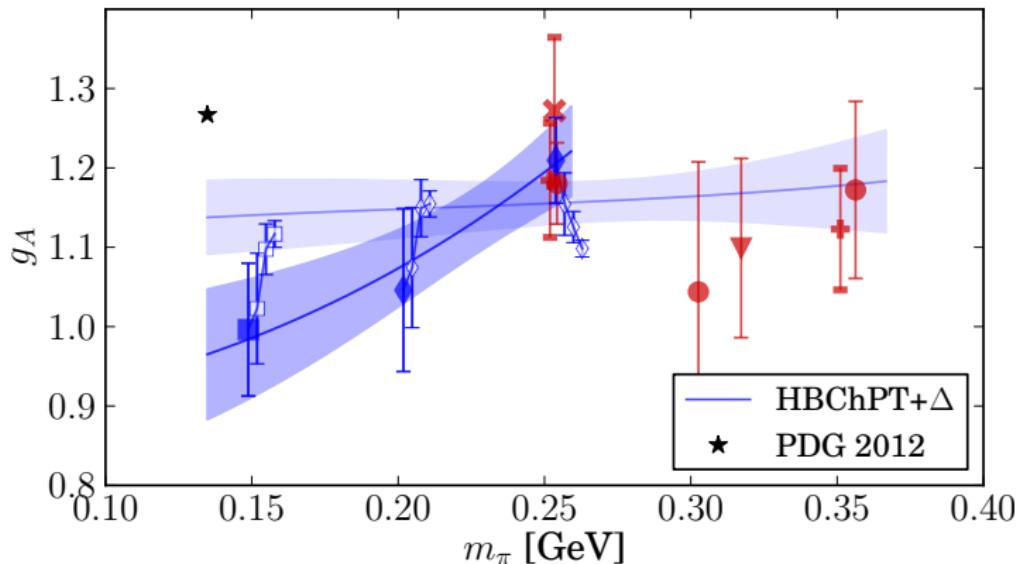


Quark Energy-Momentum: $\langle p | \bar{q} \gamma_{\{\mu} D_{\nu\}} q | p \rangle = \langle x \rangle_q \bar{u}_p \gamma^{\{\mu} p_{\nu\}} u_p$

- Measured in Deep Inelastic (e, p), (e, n) scattering:

$$\langle x \rangle_q = A_{20}^q(0) = \int dx x (q(x) + \bar{q}(x))$$

Nucleon Axial Charge g_A



$$\langle p | \bar{q} \gamma^\mu \gamma^5 q | p \rangle = g_A \bar{u}_p \gamma^\mu \gamma^5 u_p$$

- neutron β -decay, weak boson couplings
- Experimental value $g_A = 1.2701(25)$

Quark Momentum and Angular momentum

Quark energy-momentum tensor $T_q^{\mu\nu}$

$$T_q^{\mu\nu} = \bar{q} \left[\gamma^{\{\mu} i \overset{\leftrightarrow}{D}^{\nu\}} - \langle \text{trace} \rangle \right] q$$

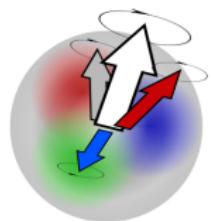
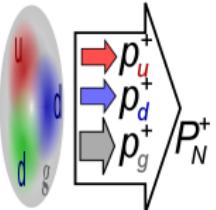
$$\langle N(P') | T_q^{\mu\nu} | N(P) \rangle \longrightarrow \{A_{20}, B_{20}, C_2\}(Q^2)$$

- quark momentum fraction

$$\langle x \rangle_q = A_{20}^q(0)$$

- quark angular momentum [X. Ji '97]:

$$J_q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)]$$



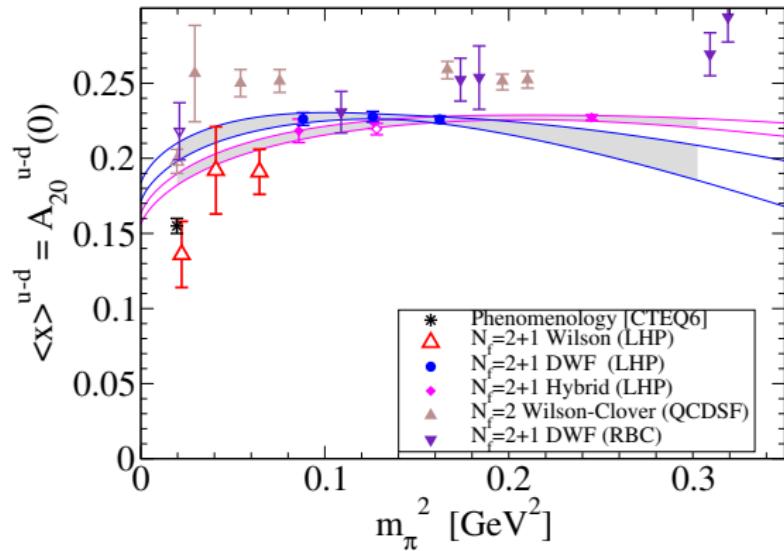
Separating contributions to nucleon spin:

- quark spin $S_q = \frac{1}{2} \Sigma_q = \frac{1}{2} \langle 1 \rangle_{\Delta q}$

- quark orbital angular momentum $L_q = J_q - \frac{1}{2} \Sigma_q$

- gluons : the rest $J_{\text{glue}} = \frac{1}{2} - \frac{1}{2} \Sigma_q - L_q$

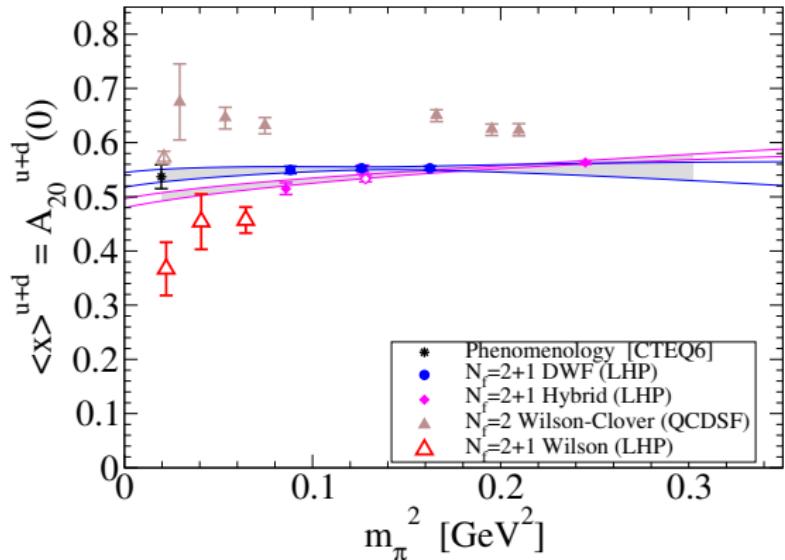
Isovector ($u - d$) Quark Momentum Fraction $\langle x \rangle_{u-d}$



$$\begin{aligned}\langle x \rangle_q &= A_{20}^q(0) \\ &= \int dx x (q(x) + \bar{q}(x))\end{aligned}$$

- “benchmark” quantity for Lattice QCD
- (long standing) problem is resolved by controlling nucleon excited states

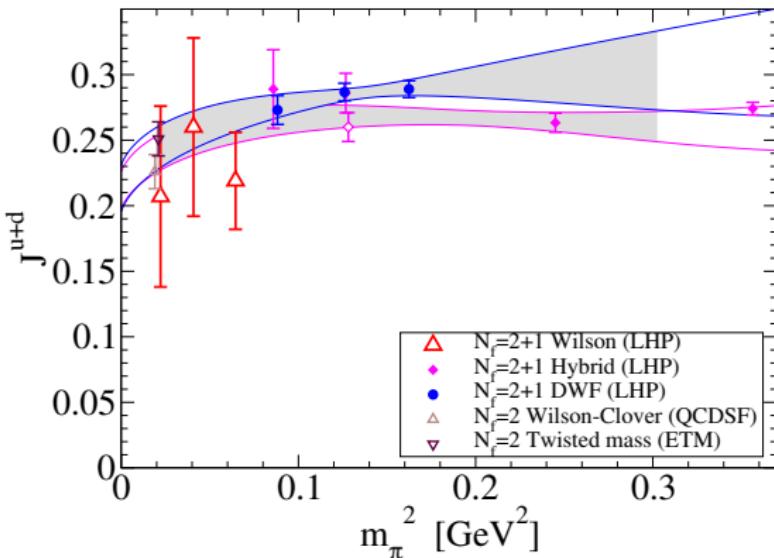
Isoscalar ($u + d$) Quark Momentum Fraction $\langle x \rangle_{u+d}$



$$\begin{aligned}\langle x \rangle_q &= A_{20}^q(0) \\ &= \int dx x (q(x) + \bar{q}(x))\end{aligned}$$

- physical point calculations are below the phenomenological value;
Disconnected Contractions?
- disagreement with [QCDSF] is likely due to excited nucleon states

Quarks Angular Momentum (1): J^{u+d}



Following [X. Ji PRL '97],

$$J_q^3 = \langle N | \int d^3x M^{012} | N \rangle,$$

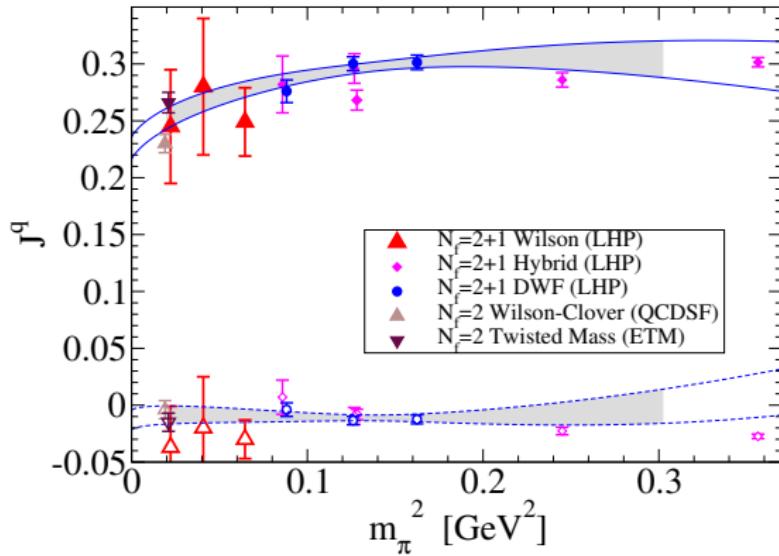
$$M^{\alpha\mu\nu} = x^\mu T_q^{\alpha\nu} - x^\nu T_q^{\alpha\mu}$$

and

$$J^q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)]$$

- Gluon contribution $J^g = \frac{1}{2} - J^q \sim 52\%$ of the nucleon spin
- result agrees with QCD sum rule estimations [Balitsky, Ji (1997)]

Quarks Angular Momentum (2): J^u , J^d



Following [X. Ji PRL '97],

$$J_q^3 = \langle N | \int d^3x M^{012} | N \rangle,$$

$$M^{\alpha\mu\nu} = x^\mu T_q^{\alpha\nu} - x^\nu T_q^{\alpha\mu}$$

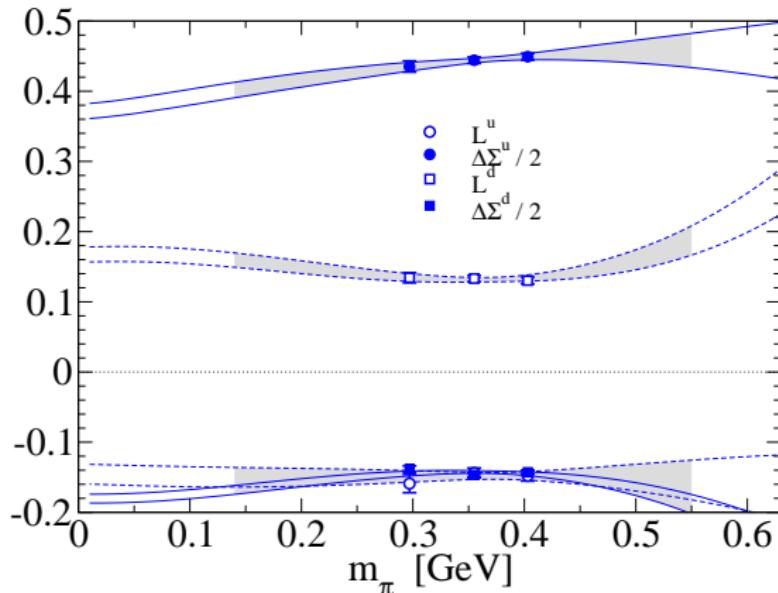
and

$$J^q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)]$$

Most contribution to the nucleon spine comes from u -quarks:

$$|J^d| \ll |J^u|$$

Quark Spin and OAM



$$L^q = J^q - S^q,$$

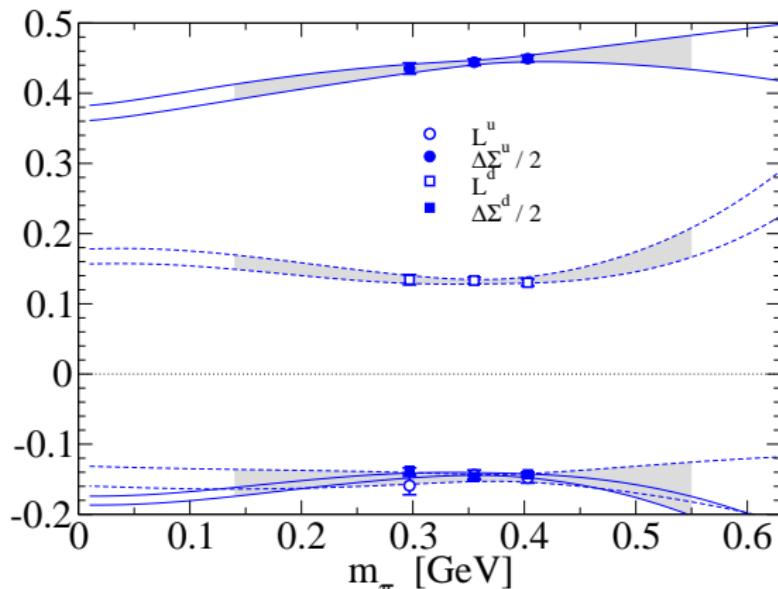
$$\begin{aligned} S_q &= \frac{1}{2} \Delta \Sigma_q \\ &= \int dx (\Delta q(x) + \Delta \bar{q}(x)) \end{aligned}$$

$$|J^d| \ll |S^d|, |L^d|$$

$$|L^{u+d}| \ll |L^u|, |L^d|$$

In agreement with [Hägler *et al* (2007)]

Quark Spin and OAM



$$|J^d| \ll |S^d|, |L^d|$$

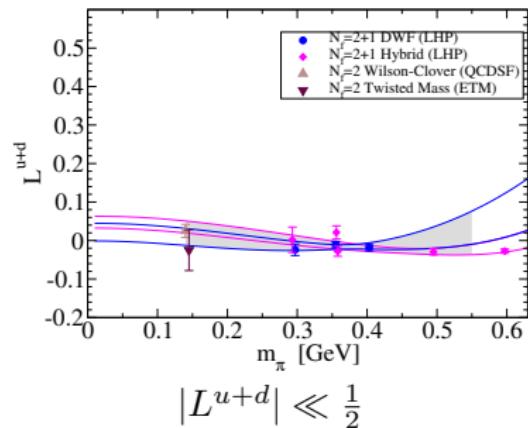
$$|L^{u+d}| \ll |L^u|, |L^d|$$

In agreement with [Hägler *et al* (2007)]

$$L^q = J^q - S^q,$$

$$S_q = \frac{1}{2} \Delta \Sigma_q$$

$$= \int dx (\Delta q(x) + \Delta \bar{q}(x))$$



$$|L^{u+d}| \ll \frac{1}{2}$$

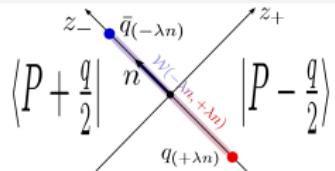
Summary

- Finally, calculations of nucleon structure near the physical pion mass (still, have to go from $m_\pi = 149(2)$ MeV $\rightarrow 134.8$ MeV)
- Four benchmark quantities agree with experiment/phenomenology:
$$\begin{aligned}\delta(\langle r_1^2 \rangle^{u-d}) &\approx 5\% \\ \delta(\langle r_2^2 \rangle^{u-d}) &\approx 10\% \\ \delta(\kappa^{u-d}) &\approx 10\% \\ \delta(\langle x \rangle_{u-d}) &\approx 15\%\end{aligned}$$
- Unresolved problem with the axial charge g_A ; thermal & finite volume effects are unlikely
- $\langle x \rangle_{u-d}, J_u, J_d$ agree with phenomenology ;
 $\langle x \rangle_{u+d}$ may be missing disconnected contributions
- Generalized Form Factors are noisy, require more statistics

BACKUP SLIDES

Generalized form factors: moments of GPDs

Generalized Parton Distributions
probe quarks with



$$\mathcal{O}(x) = \int \frac{d\lambda}{2\pi} e^{ix(2\lambda n \cdot P)} \bar{q}_{(-\lambda n)} \left[\not{\gamma} \mathcal{W}(-\lambda n, \lambda n) \right] q_{(\lambda n)}$$

$$\langle P + q/2 | \mathcal{O}(x) | P - q/2 \rangle = \bar{u}_{P+q/2} \left[\mathcal{H}(x, \xi, q^2) \not{\gamma} + \mathcal{E}(x, \xi, q^2) \frac{i\sigma^{\mu\nu} n_\mu q_\nu}{2m} \right] u_{P-q/2}$$

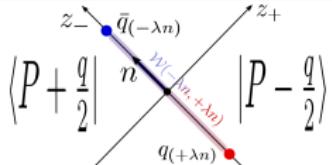
Mellin moments of \mathcal{O} are local, may be computed on a lattice :

$$\mathcal{O}_n = \int x^{n-1} dx \mathcal{O}(x) \longrightarrow \bar{q} \left[\not{\gamma} (i \overset{\leftrightarrow}{D} \cdot n)^n \right] q = \mathcal{O}_{\{\mu_1 \dots \mu_n\}} n_{\mu_1} \cdots n_{\mu_n}$$

$$\mathcal{O}_{\{\mu_1 \dots \mu_n\}} = \bar{q} \left[\gamma_{\{\mu_1} i \overset{\leftrightarrow}{D}_{\mu_2} \cdots i \overset{\leftrightarrow}{D}_{\mu_n\}} - \langle \text{traces} \rangle \right] q$$

Generalized form factors: moments of GPDs

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$$\mathcal{O}(x) = \int \frac{d\lambda}{2\pi} e^{ix(2\lambda n \cdot P)} \bar{q}_{(-\lambda n)} \left[\not{\epsilon} \mathcal{W}(-\lambda n, \lambda n) \right] q_{(\lambda n)}$$

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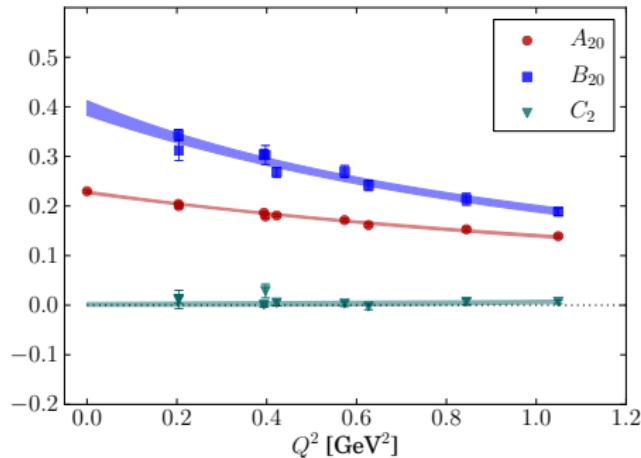
GPDs $\mathcal{H}(x, \xi, q^2)$, $\mathcal{E}(x, \xi, q^2)$ are reduced to Generalized Form Factors

$$\int x^{n-1} dx \mathcal{H}(x, \xi, q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2i} A_{n,2i}(q^2) \quad [+ (2\xi)^n C_n(q^2), \text{ even } n],$$

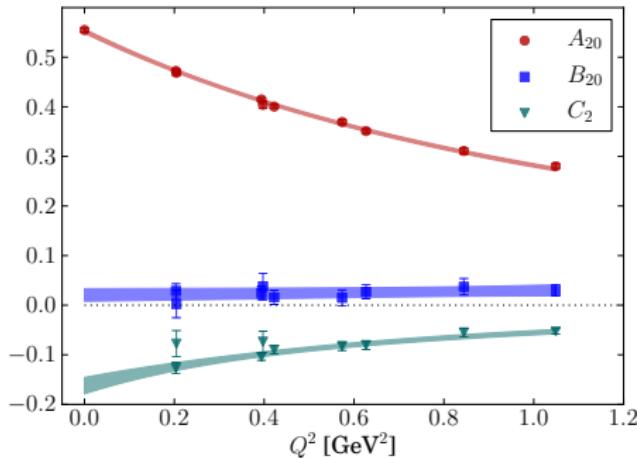
$$\int x^{n-1} dx \mathcal{E}(x, \xi, q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2i} B_{n,2i}(q^2) \quad [- (2\xi)^n C_n(q^2), \text{ even } n],$$

Unpolarized $n = 2$ GFFs

isovector $u - d$



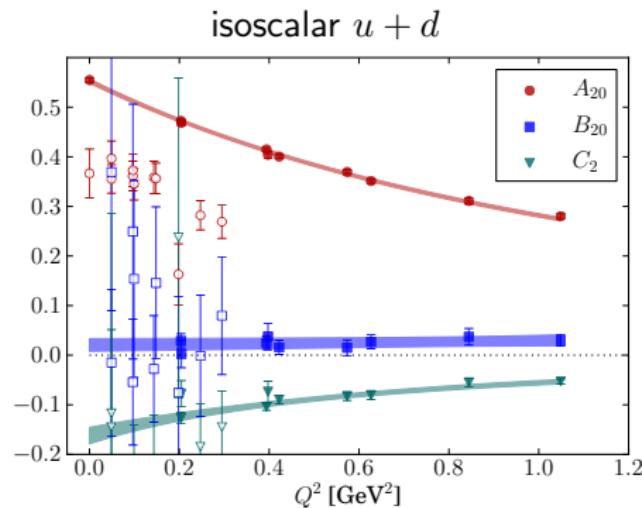
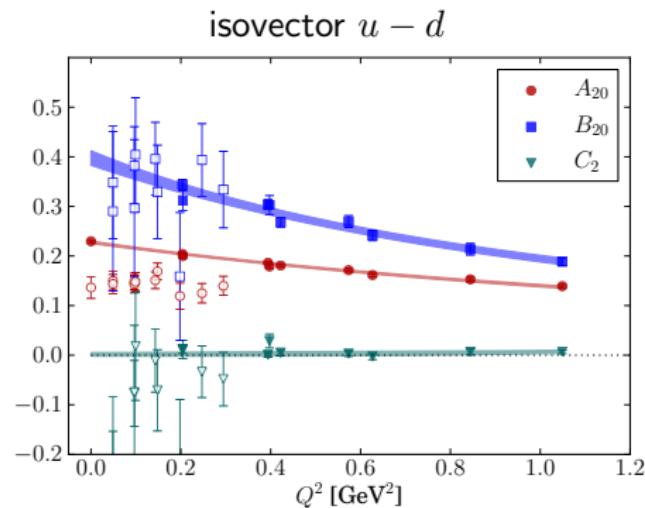
isoscalar $u + d$



[LHP collaboration]

- $m_\pi = 350$ MeV;
- $|C_2^{u-d}| \approx 0$: no ξ -dependence in the isovector channel
- large- N_c counting hierarchy holds:
 $|A_{20}^{u+d}| \gg |A_{20}^{u-d}| (\sim N_c^2, N_c)$, $|B_{20}^{u-d}| \gg |B_{20}^{u+d}| (\sim N_c^3, N_c^2)$,
 $|C_2^{u+d}| \gg |C_2^{u-d}| (\sim N_c^2, N_c)$

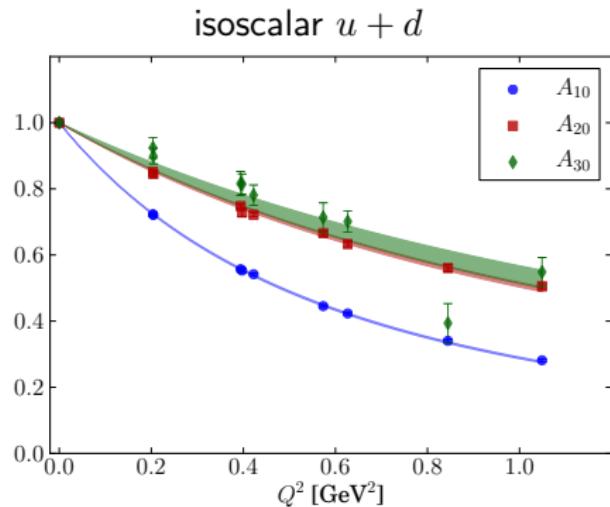
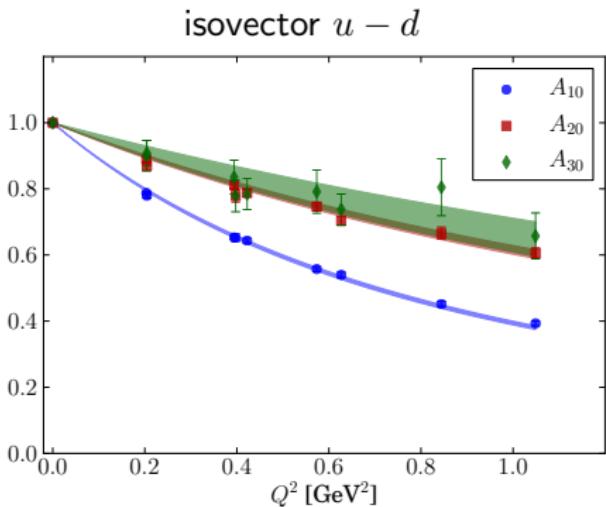
Unpolarized $n = 2$ GFFs



[LHP collaboration]

- $m_\pi = 350$ MeV; open symbols: $m_\pi \approx 150$ MeV
- $|C_2^{u-d}| \approx 0$: no ξ -dependence in the isovector channel
- large- N_c counting hierarchy holds:
 $|A_{20}^{u+d}| \gg |A_{20}^{u-d}| (\sim N_c^2, N_c)$, $|B_{20}^{u-d}| \gg |B_{20}^{u+d}| (\sim N_c^3, N_c^2)$,
 $|C_2^{u+d}| \gg |C_2^{u-d}| (\sim N_c^2, N_c)$

Comparison of unpolarized $n = 1, 2, 3$ GFFs



$m_\pi \approx 350$ MeV [LHP collaboration]

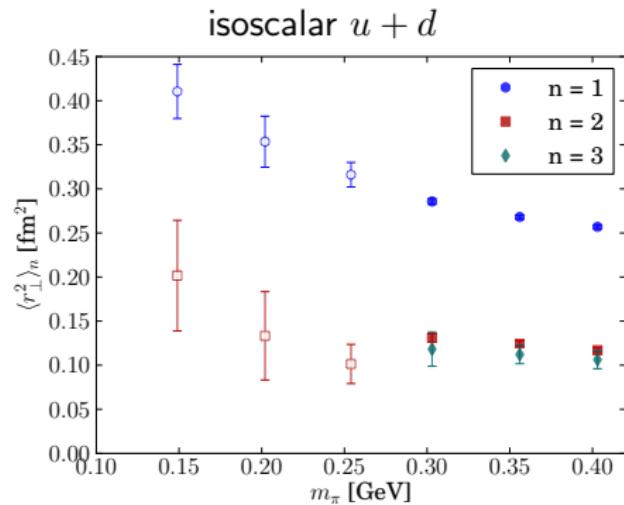
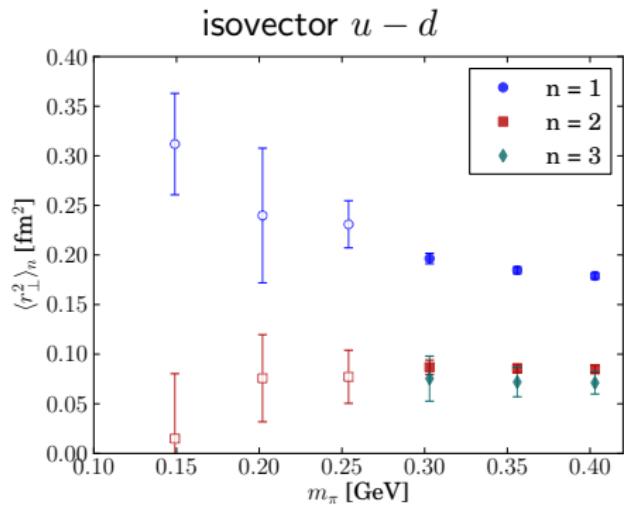
Slope of A_{n0} form factors gives transverse radii,

$$A_{n0}(Q^2) = A_{n0}(Q^2) \left[1 - \frac{1}{4} Q^2 \langle r_{\perp,n}^2 \rangle \right]$$

$$\langle r_{\perp,n}^2 \rangle = \int d^2 \vec{b}_\perp (\vec{b}_\perp)^2 \int x^{n-1} dx \mathcal{H}(x, \vec{b}_\perp)$$

$$\langle r_{\perp,1}^2 \rangle > \langle r_{\perp,2}^2 \rangle \gtrsim \langle r_{\perp,3}^2 \rangle$$

Comparison of unpolarized $n = 1, 2, 3$ GFFs



$m_\pi \approx 150 \dots 400$ MeV [LHP collaboration]

Slope of A_{n0} form factors gives transverse radii,

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$$\langle r_{\perp,n}^2 \rangle = \int d^2 \vec{b}_\perp (\vec{b}_\perp)^2 \int x^{n-1} dx \mathcal{H}(x, \vec{b}_\perp)$$

$$\langle r_{\perp,1}^2 \rangle > \langle r_{\perp,2}^2 \rangle \gtrsim \langle r_{\perp,3}^2 \rangle$$